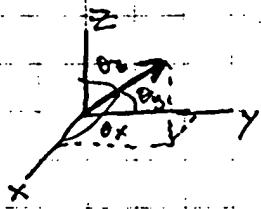


In this lattice picture a ray corresponds to ray ending exactly on lattice position remote from its origin position.

- ① pick a direction vector
- ② calculate reflection off 3 sets of orthogonal planes
- ③ calculate travel distance between successive hits on a single set of planes
- ④ assign a loss/cm number

This looks amenable to an analytic treatment.

use direction cosines to parameterize ray direction

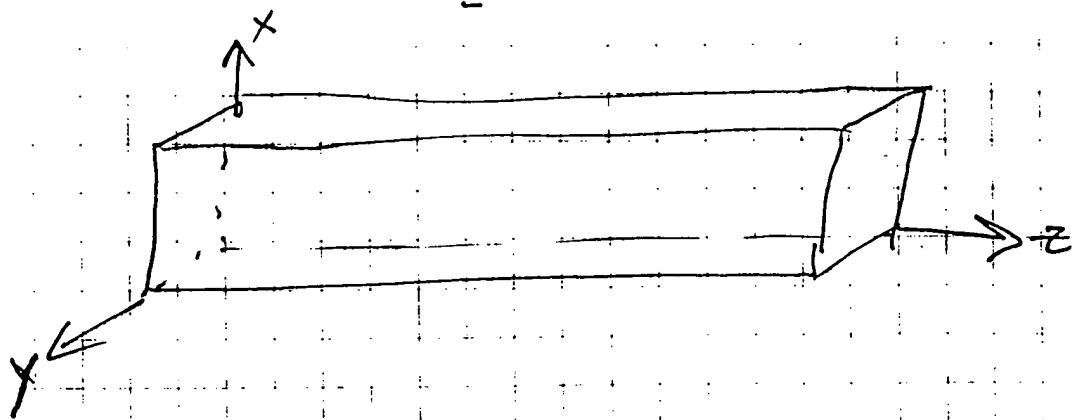


$$(\cos \theta_x, \cos \theta_y, \cos \theta_z) = \frac{(\text{RND}_1, \text{RND}_2, \text{RND}_3)}{\sqrt{\text{RND}_1^2 + \text{RND}_2^2 + \text{RND}_3^2}}$$

let Δx , Δy , and Δz denote slab dimensions or plane spacing

6-Sep-95

From this point of view it doesn't matter what position a ray is launched from, only its direction, because launch position has no impact on spacing between plan strikes.



Treat cants on surface perturbatively

2 questions:

(1) How big do cants have to be to eliminate all parasites?

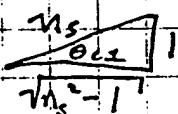
(2) For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasites.
This question will be easiest to answer for a zero loss parasitic

set z face incident angle equal to $\theta_{critz} = \sin^{-1}\left(\frac{1}{n_s}\right)$ and the x face hit and y face hit also = $\theta_{critz} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$

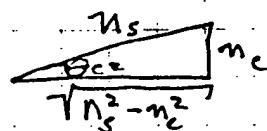
make
this
argument
more
rigorous

Now work with direction cosines

$$\theta_{cz} = \sin^{-1}\left(\frac{1}{n_s}\right)$$



$$\theta_{cz} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$$



$$\cos \theta_{cz} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos \theta_{cz} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2 \theta_{cz} + 2 \cos^2 \theta_{cz} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

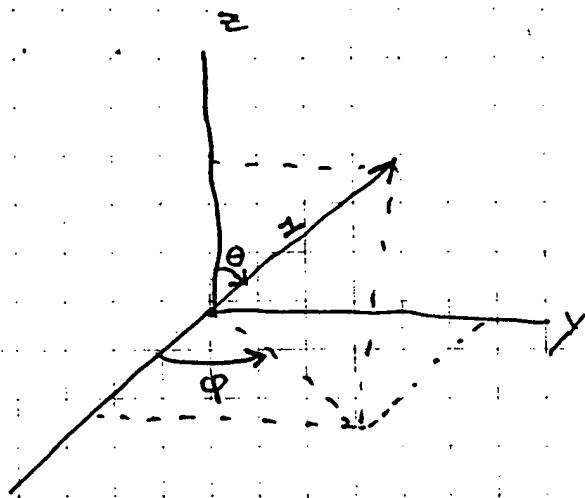
$$n_c = \sqrt{168z^2 - \frac{1}{2}} = \underline{\underline{1.677}}$$

for $n_c > 1.677$ no zero loss
passivities exist

(This agrees with
code: slab ASE 01. XCL
prediction)

Question 2 will be easiest to answer numerically:
finding the angular width over which
a passivity exists for given gain and
cladding indices.

7 - Sep - 97



$$x = \sin \theta \cos \phi = \cos \theta_x$$

$$y = \sin \theta \sin \phi = \cos \theta_y$$

$$z = \cos \theta = \cos \theta_z$$

$$\theta_x < \theta_{x\text{crit}}$$

$$\theta_y < \theta_{y\text{crit}}$$

$$\theta_z < \theta_{z\text{crit}}$$

to avoid O-loss
parasitics

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\cos \theta_x > \cos \theta_{x\text{crit}}$$

$$\cos \theta_y > \cos \theta_{y\text{crit}}$$

$$\cos \theta_z > \cos \theta_{z\text{crit}}$$

to avoid
O-loss
parasitics

$$\sin \theta_{x\text{crit}} = \frac{n_c}{n_s}$$

$$\sin \theta_{y\text{crit}} = \frac{n_c}{n_s}$$

$$\sin \theta_{z\text{crit}} = \frac{n_c}{n_s}$$

$$\cos \theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_z > \frac{\sqrt{n_s^2 - 1}}{n_s}$$

to avoid
O-loss
parasitics

$1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$ and this must be greater than

$$1 > \frac{n_c^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

to avoid
O-loss
parasitics

$$z > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$

n_s is slab index
 n_c is coating index

$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

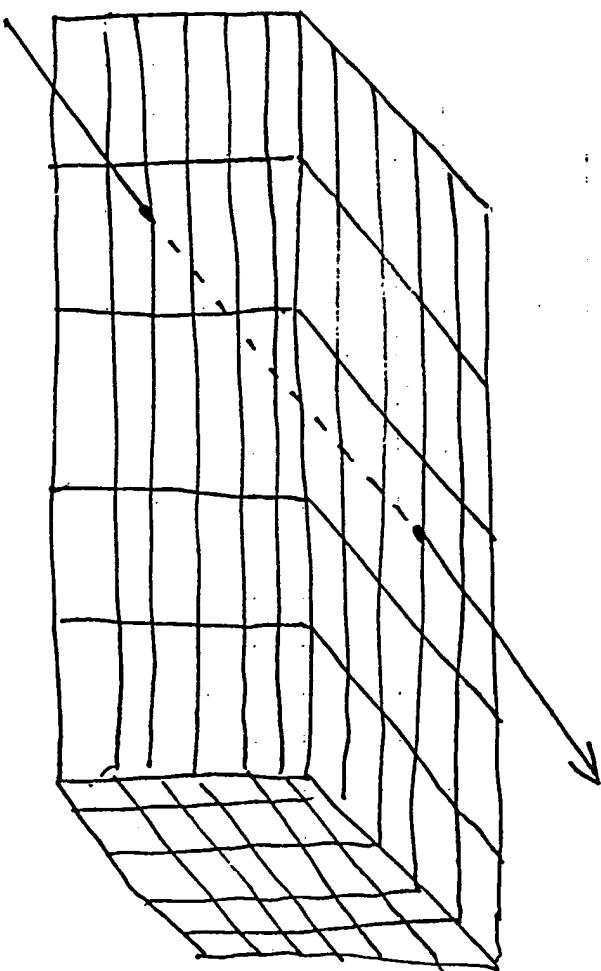
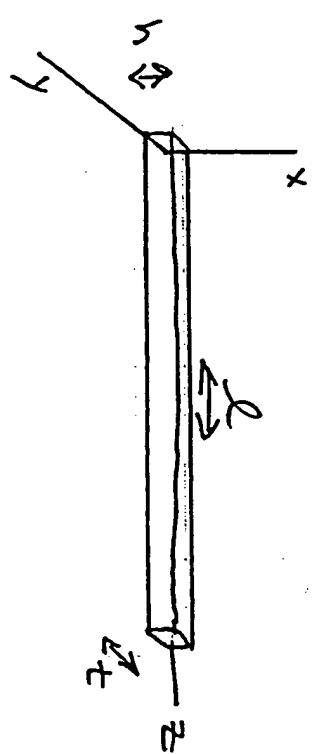
$$1 > 2(n_s^2 - n_c^2)$$

$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

using a method of images construction



fill space
using slab
and
scalar images

- Define arbitrary ray direction using direction cosines
 $(\cos \theta_x, \cos \theta_y, \cos \theta_z)$
- Gain of ray in given medium can be given by
 ~~$\frac{L_n(\text{Ref}x)}{\text{Ref}x}$~~ $\frac{L_n(\text{Ref}y)}{\text{Ref}y}$ & $\frac{L_n(\text{Ref}z)}{\text{Ref}z}$

$$G = \frac{(n/\cos_x)}{(t/\cos_y)} \quad \frac{(t/\cos_y)}{(L/\cos_z)}$$

where:
 Ref_i is the reflection coefficient for i-oriented planes
 $\frac{L}{n}$ is scalar specifying gain along

zero-loss parameters correspond to those may directions that are confined by TIR at all three sets of planes

$$\left. \begin{array}{l} \cos \theta_x < \cos \theta_{x\text{-unit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_y < \cos \theta_{y\text{-unit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_z < \cos \theta_{z\text{-unit}} = \frac{\sqrt{n_s^2 - 1}}{n_s} \end{array} \right\}$$

where:
 n_s = slab index
 n_c = coating index

• since $1 = \cos \theta_x^2 + \cos \theta_y^2 + \cos \theta_z^2$ \approx zero loss parameters exist

when $1 < \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$

or

$$n_c < \sqrt{n_s^2 - 1}$$

zero-loss parameters can be completely suppressed by choosing a cladding with refractive index large enough

$$n_c > \sqrt{n_s^2 - 1}$$